Sparse Model Identification of a 4x4 MIMO Channel Measurements in 5 GHz Band

A Degree Thesis

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by

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In partial fulfilment

of the requirements for the degree in

TELECOMMUNICATIONS SYSTEM ENGINEERING

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Abstract

The Least Absolute Shrinkage and Selection Operator has gained attention in the applied mathematics and signal processing communities.

The thesis provides us theoretical expressions for solving Compressive Sensing by the LASSO algorithm for Direction of Arrival estimation.

The central idea is highlight the fundamental concepts of the complex Least Absolute Shrinkage and Selection Operator (c-LASSO) and give an overview of its application to the Direction Of Arrival estimation.The role of the regularization parameter and suggestions on the selection are exposed. It is found that LASSO can be compared with Conventional beamforming and Least Squares.

The presented results in the context of Direction of Arrival (DOA) single snapshot estimation using a 4 antenna linear sensor array.

Resumen

El "Least Absolute Shrinkage and Selection Operator" ha ganado mucha atención últimamente en las comunidades de procesamiento de señal y del mundo de las matemáticas aplicadas.

La tesis describe de forma rigurosa la teoría y plantea al "Least Absolute Shrinkage and Selection Operator" como solución al problema del "Compressive Sensing" para la estimación de la dirección de llegada de la señal.

La idea central de este proyecto es dar a conocer los conceptos fundamentales del "Least Absolute Shrinkage and Selection Operator" y dar una visión general de su aplicación a la estimación de la DOA. Será expuesto el papel del parámetro de regularización y se darán sugerencias sobre su selección. El c-LASSO se comparará con el "Conventional Beamforming" y con el "Leasts Squares".

Los resultados serán presentados en el contexto de la estimación de la dirección de llegada para un conjunto de 4 sensores lineales.

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I would like to express my gratitude to my supervisor in Technische Universität (TU) Wien (Vienna University of Technology) Dr. Christoph Mecklenbräuker, who gave me the opportunity to join the group, supervised my bachelor's thesis and advised and guided my research.

Besides my supervisor, I would like to thank the rest of the members of the TU Wien for being always available to meet and resolve my doubts.

Last but not least, my sincere thanks to my supervisor in Universitat Politecnica de Catalunya (UPC) Dr. Javier Rodríguez Fonollosa, who advised and revised carefully my bachelor's thesis.

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1. Introduction

The project is carried out at the Institute of Telecommunications of Technische Universität of Wien, Austria.

The purpose of this project is to perform a sparse model identification of a 4x4 MIMO propagation channel in 5GHz band. The model identification is carried out by the complex-valued Least Absolute Shrinkage and Selection Operator.

The radio communication model is based on multipath propagation. Multiple transmit and receive antennas are used to increase the channel capacity by exploiting multipath propagation. In the channel sounding experiment, the primary goal is to measure the 4 x 4 MIMO propagation channel transfer function in the 5 GHz band and to infer the number of propagation paths and their statistics. The identification of propagation paths is implemented with the complex the Least Absolute Shrinkage and Selection Operator (c-LASSO) approach.

The transmitted signals in the channel sounding experiment are generated by a four-channel arbitrary waveform generator with 10 GHz of bandwidth. The multiple transmitter antenna elements generate the bandpass signals carrying the information to be transmitted. This signal will be modulated onto a carrier frequency known by the receiver, and before sending, it will be amplified and filtered to ensure sufficient receive power level and satisfying the spectral emission mask and other regulatory requirements in the 5 GHz band.

During the propagation, the transmitted signal is altered by channel attenuation, multipath distortion, antenna coupling and crosstalk, various noise sources and co-channel interference caused by electromagnetic signals not related to our experimental link.

The receiver antenna array receives radio waves and extracts the desired information by amplification to compensate channel attenuation and filtering to remove adjacent channel interferences. The received signals are directly sampled and quantized by a four-channel digital real- time oscilloscope with 12 GHz bandwidth. The subsequent digital demodulation process will convert the bandpass signal to low-pass signal. Generally, the received signals differ from the transmitted signals by channel dispersion and attenuation, noise and interference. For this reason, it is of interest to perform an accurate estimation of the channel for various receiver side signal processing.

1.1. Statement of purpose

The main achievements of this thesis are summarized as follows:

1. Matlab code implementation of sparse model identification based on c-LASSO.

2. Prove the regularization parameter importance in the data recuperation.

3. Build a Matlab code simulation of the wireless link with the same requirements as real wireless link implemented.

4. Analysis of the received data from the real-world 4x4 MIMO measurement with c-LASSO.

5.Compare c-LASSO with other parameter estimation techniques.

1.2. Requirements and specifications

Project Requirements

- Build high speed wireless link with low latency robust against interferences.
- Build transmitter and receiver side to achieve SNR and distance targets.
- Select accurately the parameters such us signal power, amplification stage, filters and cables.
- Receive and store channel sounder signals during the real-world 4x4 MIMO measurement.
- Build a Multi-tone (MT) Frequency Division Multiplexing (MTFDM) sounding sequence.
- Data acquisition and processing of the obtained data.

Project specifications

- Multi-tone (MT) Frequency Division Multiplexing (MTFDM) 80Mhz Bandwidth.
- Central frequency 5.725 GHz.
- 641 Tones with phase for low crest factor (Zadoff-Chu sequence).
- Tone spacing 125 KHz.
- Maximum signal power at transmitter side 13 dBm
- Channel bandwidth 80 MHz.
- Four transmit dipoles in the 5GHz band.
- Four receiver dipoles in the 5GHz band.
- Different Configurations and Orientation of the antennas at transmitter and receiver side.
- Maximum long distance 600 meters.
- Desired SNR 20 dB
- GPS Rubidium as a frequency standard for the

1.3. Background

The main project stems from the need to develop a high speed wireless link with low latency for an audiovisual company. The mobile communications department of the Institute of Telecommunications will perform the system with the accurate specifications according to the needs of the company.

My bachelor thesis is a contribution to the main project that starts from the scratch. Christoph Mecklenbräuker as my supervisor in the Technische Universität of Wien has suggested the thesis topic and requirements for the performance of a sparse model identification of a 4x4 MIMO propagation in 5GHz band.

Why Compressive Sensing? Why Least Absolute Shrinkage and Selection Operator?

The purpose of this project is to perform a sparse model identification of a 4x4 MIMO propagation channel in 5GHz band.

One straightforward approach to estimate the channel is to search the angles of departure and the angles of arrival exhaustively, and use the direction with the largest gain as the beamforming direction. The estimation accuracy of this approach is limited by the number of antennas, the paths of the transmitted signal, attenuation loss and guarantee high signal to noise ratio.

In the practical experiment, the signal paths are mostly influenced by two factors:

- Abrupt changes in the Line of Sight causes the disappearance or appearance of the dominant paths. This can be induced by environmental changes that block the signal like vehicles or pedestrians.
- Noise that affect the angle of departure (AOD) or the angle of arrival (AOA) originated by measurement devices. Other impact factor is small rotation of the transmitter or the receiver antennas.

The received signal, as a result of propagation phenomena in a radio channel, the radio waves arrive at the receiver from different directions and different amplitude, phase and time delay, this in combination is a multipath propagation.

The topic of Direction Of Arrival estimation has gained much attention in the applied mathematics.

Traditionally, DOA estimation is addressed by methods such as Conventional beamforming, the Maximum likelihood method (ML), Multiple Signal Classification method (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) .

Recently, sparsity based optimization and Compressed Sensing have attracted great interests.

A signal is called sparse if it can be exactly represented by a basis and a set of coefficients where only a certain number of them are non-zero.

The Shannon/Nyquist sampling theorem specifies that to avoid losing information when capturing a signal, the sampling rate must be at least twice the maximum frequency present in the signal. In communications, the Nyquist rate sometimes is too high that the resulting number of samples is too many to process the data.

Compressed Sensing is a novel sensing/sampling paradigm that captures and represents signals at a rate significantly below the Nyquist rate. This is a signal processing technique with a high-resolution method, even with single snapshot of the data, and works with low signal-to-noise ratio. Main inconvenience is the computational complexity and the difficult implementation due to its solution is not convex.

In this thesis we propose the complex version of the Least Absolute Shrinkage and Selection Operator as a convex solution of the Compressed Sensing.

The complex LASSO is compared with conventional methods:

Maximum likelihood techniques has been investigated for DOA estimation because it brings highly accurate DOA estimation results. Compared with the topic exposed in this thesis, the ML method requires more time and complex algorithms.

MUSIC algorithm is based on the eigenvalue decomposition of the covariance matrix of the received data.The main advantage is the easy implementation and the high resolution, but the exact number of sources is need to be known in advantage.

The ESPRIT algorithm uses the shift-invariance properties of the receiver array signal taking into consideration that the array is composed of two identical subarrays. However, this implementation also uses the eigenvalue decomposition and requires the number of sources to be known in advanced like the MUSIC Algorithm.

MUSIC and ESPRIT, both compared with the c-LASSO , the DOA estimation can be done without a priori knowledge of the number of paths that arrive at the receiver side.

Especially used for low signal-to-noise ratios, spatially-closed sources and coherent scenarios.

1.4. Work plan with tasks, milestones and Gantt diagram

Project: **Literature search**	WP ref: **WP0**	
Major constituent: Research		
Short description: Become comfortable with the wireless communication MIMO standard. Gather information, understand and comprehend the complex-LASSO as a sparse signal estimator.	Planned start date: 06/03/2018 Planned end date: 28/06/2018 Start event: 06/03/2018 End event: 28/06/2018	
Internal task T1: Set the project goals and organize the tasks. Internal task T2: Background investigation of compressed sensing and c-LASSO.	Deliverables: NA	Dates: NA

Table 1

Project: **Channel Measurements**	WP ref: **WP1**	
Major constituent: hardware prototype of the wireless link		
Short description: The wireless link will be built firstly in the laboratory. Preparation of the equipment and instruments involved in the experient to perform the measurements in Seestadt, Vienna.	Planned start date: 03/04/2018 Planned end date: 06/04/2018 Uni roof top : 09/05/2018 Seestadt : 29/06/2018	
Internal task T1: Define the measurement parameters. Internal task T2: Measurements performed in the University roof. Internal task T3: Measurements performed in Seestadt.	Deliverables: T1: Reporting T2 T3	Dates: 25/04/2018 09/05/2018 27/05/2018

Table 2

Project: **Matlab implementation of sparse model identification**	WP ref: **WP2**	
Major constituent: Simulation		
Short description: Matlab algorithm of perfect model identification on synthetic 4x4 MIMO simulation data. The code must identify the Low Mean Squared Error of the residuals after model identification.	Planned start date: 05/04/2018 Planned end date: 31/05/2018 Start event: 05/04/2018 End event: 31/05/2018	
Internal task T1: First code version. Internal task T2: Validation of the proper functioning of the Matlab code.	Deliverables: Reporting	Dates: 31/05/2018

Table 3

Project: **Analysis of selected data from the channel measurements**	WP ref: **WP3**	
Major constituent: Simulation		
Short description: Analysis of selected data sets from WP1 with the Matlab code performed.	Planned start date: 31/05/2018 Planned end date: 25/06/2018 Start event:29/05/2018 End event: 29/05/2018	
Internal task T1: Draw conclusions about the c-LASSO estimator.	Deliverables: Reporting	Dates: 28/06/2018

Table 4

Project: **Documentation**	WP ref: **WP4**	
Major constituent: Thesis writing		
Short description: Draft all the documentation required in the final bachelor thesis.	Planned start date: 06/03/2018 Planned end date: 28/06/2018 Start event: 06/03/2018 End event: 28/06/2018	
Internal task T1: Project proposal and work plan Internal task T2: Project critical review Internal task T3: Final Report Template	Deliverables: Documentation	Dates: Depending on the document.

Table 5

Milestones

WP#	Task#	Short title	Milestone / deliverable	Date
4	1	PP and WP	Project Proposal and Work Plan	09/04/2018
1	1	Channel measurements	Define Parameters	25/04/2018
4	2	PCR	Project critical review	08/05/2018
2	1	Matlab algorithm	First Test	14/05/2018
1	2	Channel measurements	Measurements in the university	09/05/2018
1	3	Channel measurements	Measurements in Seestadt	29/05/2018
2	2	Matlab algorithm	Validation Matlab code	31/05/2018
3	1	Analysis of data	Draw conclusions	28/06/2018
4	3	FRT	Final Report Template	28/06/2018

Table 6

Time plan

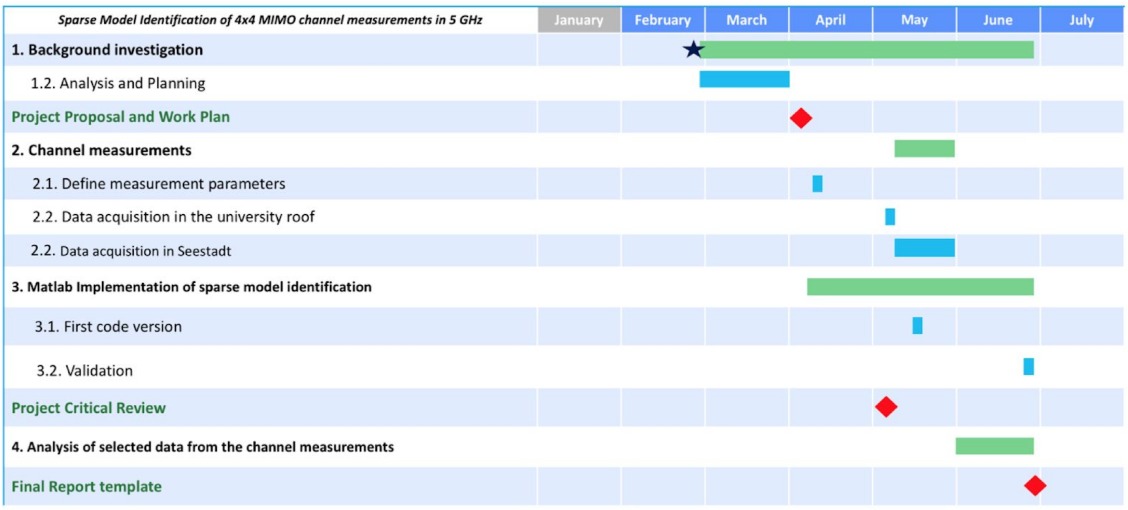


Table 7

1.5. Deviations from the initial plan and incidences

The incidences can be summed in distinguishing between the major project where I am involved in and my bachelor thesis.

It is necessary to explain the incidences in the major project in chronological order.

The generation of the test signals has lasted for longer than the deadlines proposed. This signals are needed to prove if the receiver part of the wireless link is functioning properly.

In the meanwhile, other members of the group faced problems with the electronic instrumentation, such as the batteries for power supply of transmitter and receiver sides.

Once implemented the test signals, the theoretical calculations have been demonstrated that signal-to-noise ratio is lower than we had expected. To address this problem, some solutions have been proposed like increase the power of the transmitted signals, but we have a trade off between the input power signal and the amplifiers. There is no chance to increase the power because the amplifiers must operate at linear region to avoid distortion.

The alternative solution is to change the dipoles for other ones optimized to work in the 5GHz frequency range that will bring more gain to signal.

The gain on the transmitter side of the communication has been improved, which is expected to solve significantly the signal-to-noise ratio at the receiver side.

The incidences of my thesis has been solved thanks to my supervisor. At the beginning, I had misunderstood the complex Least Absolute Shrinkage and Selection Operator approach that I have solved with papers and books about estimation and optimization.

Other difficulties that I had faced during the implementation of the Matlab code are the following. Before the channel estimation with the LASSO approach I had to build test complex-valued signals as similar as possible to the signals used on the major project.

Finally, I have faced some problems to understand how to approach the Matlab code implementation of the complex version of the LASSO.

2. State of the art of the technology used or applied in this thesis:

2.1.Notation

We represent matrices with upper-case $A, B \ldots$ and vectors $a, b \ldots$ with lower case letters. The inverse, the transpose and the Hermitian transpose are denoted as X^{-1}, X^T, X^H.
Let define the Moore-Penrose pseudo inverse as $X^+ = (X^H X)^{-1} X^H$.

The l_p-norm of a vector complex valued $x \in C^N$ is defined as $\|x\|_p = \left(\sum_{n=1}^{N} |x_n|^p \right)^{\frac{1}{p}}$.

By extension, the l_0-norm is defined as $\|x\|_0 = \sum_{n=1}^{N} 1_{x_n \neq 0}$ is the counting function that return the number of non-zero elements of the vector, and the l_∞-norm as $\|x\|_\infty = max_{1 \leq n \leq N} |x_n|$.

2.2. Single Snapshot DOA Estimation Signal Model

It is now assumed that in a time instant *k,* there is $x_k = (x_1, \ldots, x_M)^{\mathbf{T}}$ that is a complex-valued source vector with the amplitudes values. Where M is the number of possible source directions, the Direction Of Arrival angles.
The receiver sensor signals at each k instant, is denoted as $y_{k1}(f_j) = (y_{k1}(f_j), \ldots, y_{kN}(f_j))^{\mathbf{T}}$. Where N is the number of array sensors in the receiver side and $j = 1, \ldots, J$ is the index of the frequency of interest in the frequency band where is proposed our system.

The linear model for the narrowband sensor array data y, which is based our system, at the frequency f_j is the following,

$$y(f_j) = A(f_j)\, x + n(f_j)$$

Eq(1)

The sensing matrix A_{nm} is composed by the steering vectors a_m with the hypothetical waves from DOA θ_m,

$$A = \left[a(\theta_1), \ . \ . \ . \ , \ a(\theta_m)\ \right]$$

Eq(2)

$$A_{nm}(\ f_j\ ) = \frac{1}{\sqrt{N}}\left[1,\ e^{-j2\pi f_j\,(1)\,d\,c^{-1} sin(\theta_m)} \quad \ldots,\ e^{-j2\pi f_j\,(N-1)\,d\,c^{-1} sin(\theta_m)}\right]$$

Eq(3)

The steering vector described represents the propagation delay from the mth potential narrow bands emitted simultaneously to each N array sensors. In other words, we model a uniform array (ULA), wich is described with its steering vectors representing the incident wave for each array element.

The A_{nm} matrix must have the restricted isometry property, that allow A_{nm} to be nearly-orthonormal when the equation system works with sparse vectors.

Modeling an uniform array (ULA) we must define the sensor spacing $d = \frac{\lambda}{2}$, where λ is the wavelength for the frequency f_j , the speed of wave propagation denoted by c.

As said before, the vector θ_m is the result of discretizing the half-space of interest $\theta\ \in [-90^o,\ 90^o\ ]$ with the following formula $\theta_m\ = \frac{(m-1)180^o}{M} - 90^o$.

The additive random noise vector $n(f_j)$ at the j th frequency band. The noise is considered spatially uncorrelated in the half-space of interest and follows a zero-mean complex normal distribution.

The signal-to-noise ratio vector, for a single snapshot is defined as,

$$SNR = 10log_{10}\frac{E\left\{\|Ax\|_2^2\right\}}{E\left\{\|n\|_2^2\right\}} \quad \text{(dB)}$$

Eq(4)

2.3. Problem formulation

The thesis central idea is to solve DOA estimation with the Compressive Sensing framework. Compressed Sensing is a technique for signal reconstruction, where the underlying linear model that relates measures and the searched parameter consists of an undetermined system of equations. It has been applied in various areas, but recently has gained much attention in the applied mathematics and signal processing communities.

In communications, compressive sensing is largely accepted for sparse channel estimation, due to the multipath channels are sparse in their equivalent baseband representation. The central idea behind the CS is knowing a priori that the signal vector to be recovered is sparse. In other words, most of the entries of this signal vector are null.

The problem formulation for Compressed sensing is applied to DOA single snapshot estimation with the $y \in C^N$ as the complex array data at the receiver sensors and array steering matrix $A_{nm} \in C^{NxM}$. The goal is to find a sparse signal $x \in C^M$ for given spartisy order $K_0 \in N$, where $K_0 \ll N\ observation\ sources$, that minimizes the squared data residuals.

$$\widehat{x}_{l_0} = argmin_{x \in C^M} \| y - Ax \|_2^2 \quad subject\ to \quad \| x \|_0 \leq K_0$$

Eq(5)

As a result that we can express DOA estimation as a linear undetermined problem with a sparsity constraint. However, the x_{l_0} is known as a complex-valued l_0 - reconstruction that is non-convex and computationally hard to solve.

We point out the solution in [6] that explains how with the Compressed Sensing theory and satisfying the requirements such as our signals are sparse enough ($K_0 \ll N\ observation\ sources$), and the sensing matrix has incoherent columns,

we can assume that l_0 - norm minimization problem *Eq(5).* is equivalent to the l_1 - norm minimization problem at least in the noiseless case.

Thus, the problem formulation in *Eq(1).* involves noisy measurements, it fits with the definition above mentioned. Hence, we can define the estimation of $\widehat{x}_{l_1}$ with the l_1 - norm minimization problem as,

$$\widehat{x}_{l_1}(\varepsilon) = argmin_{x \in C^M} \ \| x \|_1 \ \ subject\ to \ \ \| y - Ax \|_2 \ \le \varepsilon$$

Eq(6)

The ε is the noise floor, it regularizes and limits the level of noise that our system permits for obtaining properly the data.

The l_1 - norm minimization problem can be reformulated with a regularization parameter $\mu \geq 0$ real valued . This new equation is the complex Least Absolute Shrinkage and Selection Operator.

$$\widehat{x}_{LASSO}(\mu) = argmin_{x \in C^M} \ \| y - Ax \|_2^2 \ + \mu \ \| x \|_1$$

Eq(7)

The c-LASSO is a least squares optimization with the l_2 - norm that provides an estimation of the data, while the μ parameter regularizes the sparsity level of the l_1 - norm.

Therefore, c-LASSO algorithm presents a trade-off between the sparsity of the estimated signal and the possibility to obtain the desired signal from the acquired data with low noise interference.

The algorithm enable to obtain the best data estimation by changing the value of the regularization parameter, so the main challenge is to find the proper value that solves *Eq(7).* or equivalently *Eq(6).*

The final purpose is to estimate the $\widehat{x}$ complex-valued sparse vector that has the amplitude values of distinct DOAs, which are determined by the equation,

$$\widehat{x}_{CS} = A_S^+ \ y$$

Eq(8)

The matrix $A_s^+ \ \in \ C^{NxK}$ that is only built with the steering vectors that correspond to the non-zero values from the $\widehat{x}_{LASSO}(\mu)$ estimated vector .

Given K sparsity order and S the number coefficients non-zero, if $K = S$ then the c-LASSO fins the best data fit.

$$\widehat{x}_{l_0}(K) = argmin_{\ x \in C^N} \| y - Ax \|_2 \quad subject\ to\ \|x\|_o = K$$

Eq(9)

Concluding, we are able to solve the Direction of Arrival estimation throw the Compressed Sensing technique by finding an alternative convex solution for the l_0-reconstruction, computational easy, maintaining the sparsity of signals and its incoherence. The LASSO will solve the problem of the DOA estimation.

2.4. Regularization parameter

As commented on previously, the operating principle for the proper reconstruction of the signal depends on the value of the regularization parameter, due to it controls the balance between the data and the sparsity of the solution. The most remarkable values are listed below.

As said before, *Eq(7)* is equal to *Eq(6)*. because for every different value of ε, we can find a regularization parameter μ that l_1- norm solution has the same value in both equations.

$$\widehat{x}_{l_1}(\varepsilon) = \widehat{x}_{LASSO}(\mu)$$

The c-LASSO algorithm when regularization parameter takes large values, $\mu \uparrow\uparrow$ more coefficients of the x vector become zero. The estimated signal becomes more sparse and the minimization of the l_2- norm term provides a best data fit because it has to estimate less coefficients of x. Exists a maximum threshold μ value, if this value is exceed will convert more coefficients into cero, and we take the risk of eliminate some of the significative coefficients of the signal that we want to recover.

The c-LASSO algorithm when regularization parameter takes low values $\mu \downarrow\downarrow$ the x vector is less sparse, for this reason some coefficient values that are not relevant, are detected as desired signal. This induces crosstalk in de detection.

The c-LASSO algorithm when regularization parameter takes the zero value , $\mu = 0$, is related to the Conventional Beamforming.

Conventional Beamforming is the simplest form of beamforming techniques, despite that is considered as a field of array processing. As a DOA estimation method the CBF is a spatial filter that processes the data obtained from array sensors that are weighted with fixed values. This weighted fixed values are obtained by the Least Mean Squares estimation. Values obtained create a beam structure that enables to receive the data in the desired direction while it destroys or attenuates the signals that come from other different directions caused by reflections. The significant advantages are the simplicity implementation and robustness, but it presents a limited angular resolution and it is impossible to adapt to signals that come in other direction without replacing the weights values.

The traditional methods estimate DOA with the non-sparse solution l_2 - norm regularized least squares.

$$\widehat{x}_{l_2}(\mu) = argmin_{x \in C^M} \| y - Ax \|_2^2 + \mu \| x \|_2^2$$

Eq(10)

The parameter controls the importance between the data fit and the energy of the estimated signal.

The equivalence between c-LASSO and l_2 - norm regularized least squares solution can be found reformulating LASSOs equation defined *Eq(7).*

$$\widehat{x}_{LASSO} = argmin_{x \in C^M} \| y - Ax \|_2^2 \quad subject\ to \quad \| x \|_1 < \delta_{LASSO}$$

Eq(11)

The l_2 - norm regularized least squares must be reformulated equally,

$$\widehat{x}_{Least\ Squares} = argmin_{x \in C^M} \| y - Ax \|_2^2 \quad subject\ to \quad \| x \|_2^2 < \delta_{LS}$$

Eq(12)

Thus, if the $\left\|\widehat{x}_{LS}\right\|_1 = \delta_{LS}$ and we find a Least Squares parameter $\delta_{LS} < \delta_{LASSO}$, then we conclude $\widehat{x}_{LASSO} = \widehat{x}_{Least\ Squares}$.

Combining the assumptions above mentioned we obtain,

$$\widehat{x}_{LASSO} = \widehat{x}_{Least\ Squares} = A^{+}y$$

2.5. Software development

The Matlab code implemented in this thesis and every algorithm build to resolve and understand better the complex Least Absolute Shrinkage and Selection Operator has been done by me.

The CVX Toolbox for Disciplined Convex Programing has been used which is available in the Matlab environment.

3. Project development

The purpose of this chapter is to make a general description of the complete system, followed by its specific hardware and software parts.

3.1 . System Description

The main project has the following schematic for the transmitter side.

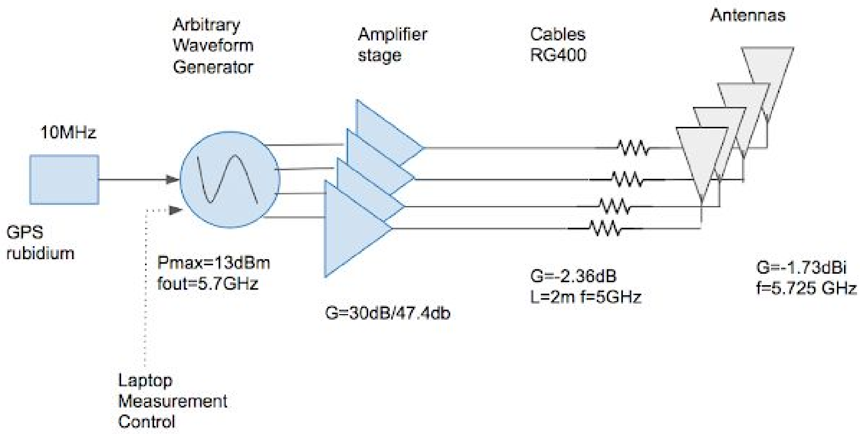
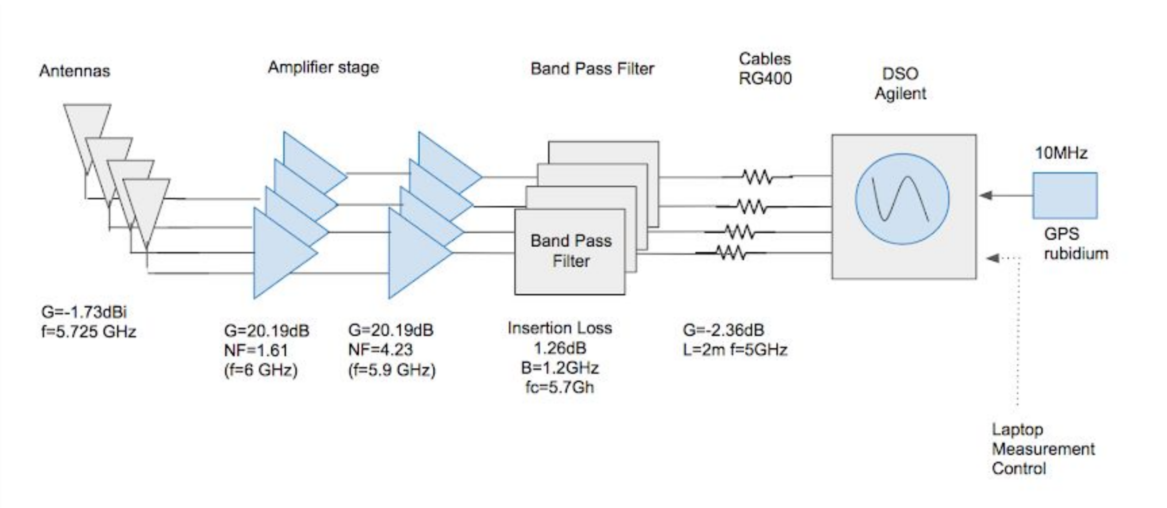


Figure1

The transmitted signals in the channel sounding experiment are generated by a four-channel arbitrary waveform generator. The MTFDM signal (Multi-tone frequency Division Multiplexing) generated is amplified before sending to ensure sufficient receive power level.

The main project has the following schematic for the transmitter side. Figure 2



The receiver antenna array receives radio waves and extracts the desired information by amplification to compensate channel attenuation and filtering to remove adjacent channel interferences.

The received signals are directly sampled and quantized by a four-channel digital real-time oscilloscope with 12 GHz bandwidth. The subsequent digital demodulation process will convert the bandpass signal to low-pass signal. Then, the signal will be stored for later process in the laboratory.

3.2. Methodology in Channel sounder measurements

The methodology developed for the measurements is based on the different types of antenna configuration and orientation. Transmitter and receiver side had to adjust to the following configurations:

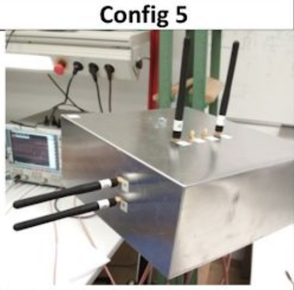
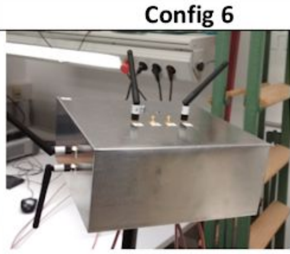
Antenna description: WSS002 Dual Band Antenna with RP-SMA(M)		
Config 1	**Config 2**	**Config 3**
 Port1 – Antenna 1 Port2 – Antenna 2 Port3 – Antenna 3 Port4 – Antenna 4 Port5 – X Port6 – X	 Port1 – Antenna 1 Port2 – Antenna 2 Port3 – Antenna 3 Port4 – Antenna 4 Port5 – X Port6 – X	 Port1 – Antenna 1 Port2 – Antenna 2 Port3 – Antenna 3 Port4 – Antenna 4 Port5 – X Port6 – X
Config 4	**Config 5**	**Config 6**
 Port1 – Antenna 1 Port2 – Antenna 2 Port3 – Antenna 3 Port4 – Antenna 4 Port5 – X Port6 – X	 Port1 – Antenna 1 Port2 – X Port3 – X Port4 – Antenna 4 Port5 – Antenna 2 Port6 – Antenna 3	 Port1 – Antenna 1 Port2 – X Port3 – X Port4 – Antenna 4 Port5 – Antenna 2 Port6 – Antenna 3

Figure 3

Orientation

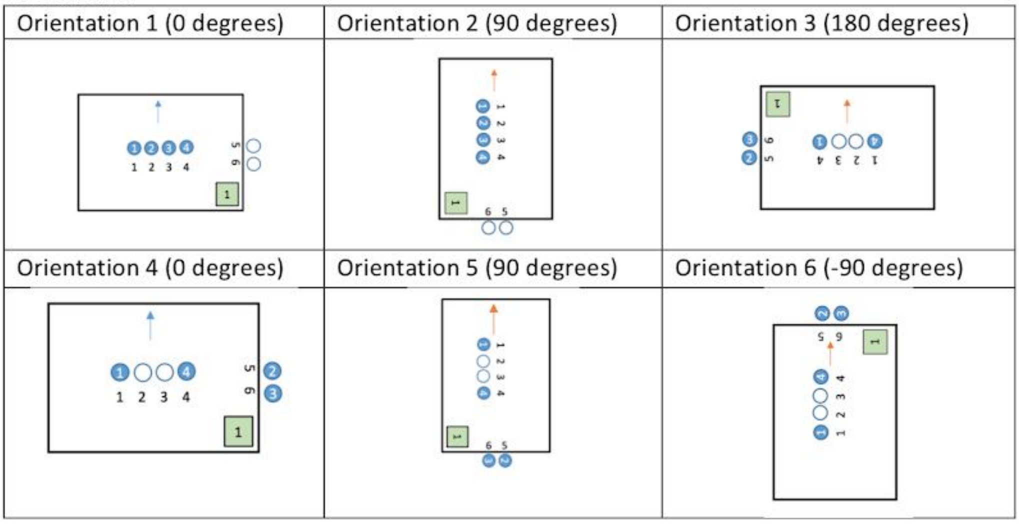


Figure 4

The purpose of performing numerous scenarios is the high diversity of data acquisition. Then in the laboratory, data processing will show the best scenarios that brings better signal-to-noise ratio.

Changes in the configuration and orientation of the antennas at both sides, cause different scenarios. Since Direction of Arrival is affected by this type of changes, later in this thesis is shown how different DOAs are estimated if we change antenna array configuration and orientation.

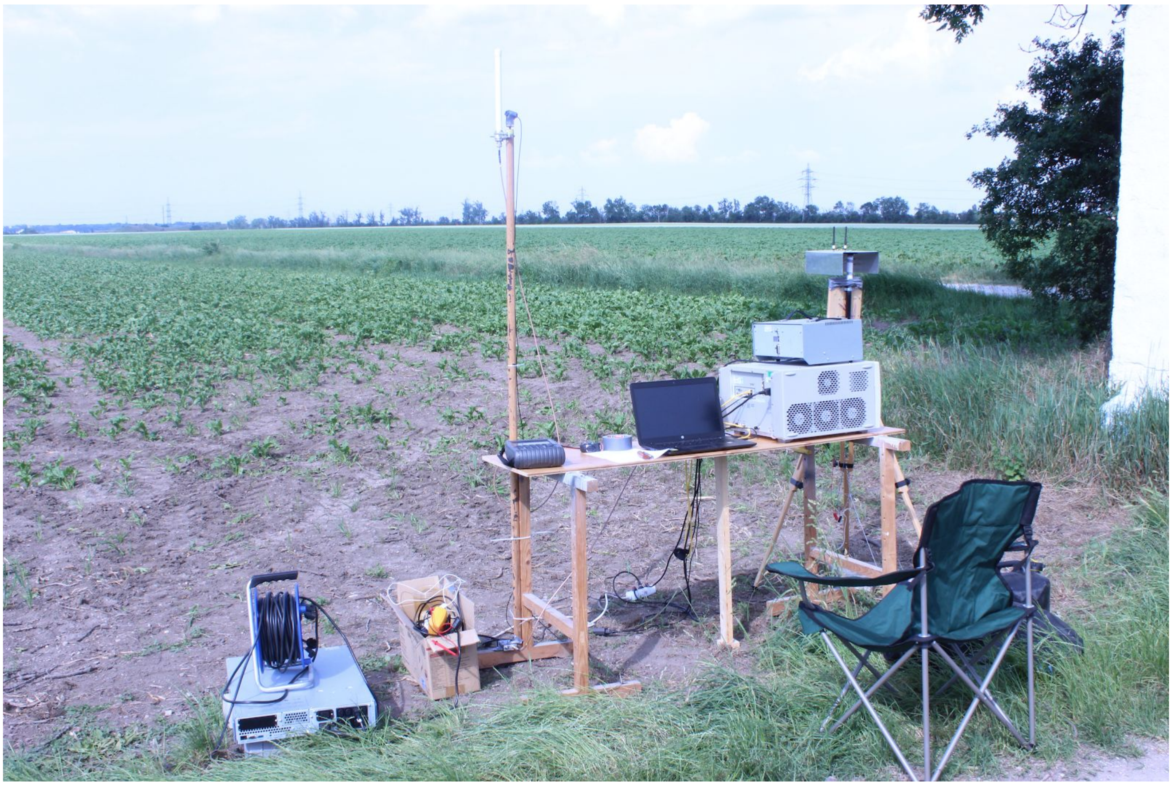
The measurement was acquired in Seestadt, Vienna in the middle of the countryside in a crop field. The location was decided there due to the necessity of having 600 meters without anything that could interfere in the Line-Of-Sight propagation.

The location contribute to generate some challenges to face, such as transport all the necessary equipment include Arbitrary waveform generator, oscilloscope and big battery packs.

The other challenge that the project faced was the necessity of having the transmitter and the receiver synchronized at the same center frequency. The solution came with the implementation, on the both sides, of a rubidium GPS. This device provides a signal in the desired central frequency. With the rubidium GPS we feed the Arbitrary waveform generator and the oscilloscope.

Figure 5. Transmitter Side

Figure 6. Receiver Side



4. Results

In this section, the performance of the proposed sparse channel identification of a 4x4 MIMO propagation in 5GHz band is evaluated based on numerical simulation. The simulation build tries to be the most accurate as possible as the main project requirements.

Synthetic data from a uniform linear array is build with N=4 elements with half-wavelength spacing, the default central frequency is 5.725 GHz. The DOA domain is discretized by $\theta_m = [-90^o : 0.5^o : 89.5^o]$ with $m = 1, ..., M$ and M=360. The simulation scenario has K=2 far-field plane wave sources that arrive to the receiver side. The two values are stationary at [45º , 26.5º] (relative to broadside) with constant power level [3, 27] dB respectively. The signal-to-noise ratio is 20dB. The noise is zero-mean complex-valued symmetric and normally distributed. All the matrix and vectors are complex valued.

4.1.Theoretical demonstrations

Theoretical demonstrations about the regularization parameter assumptions.

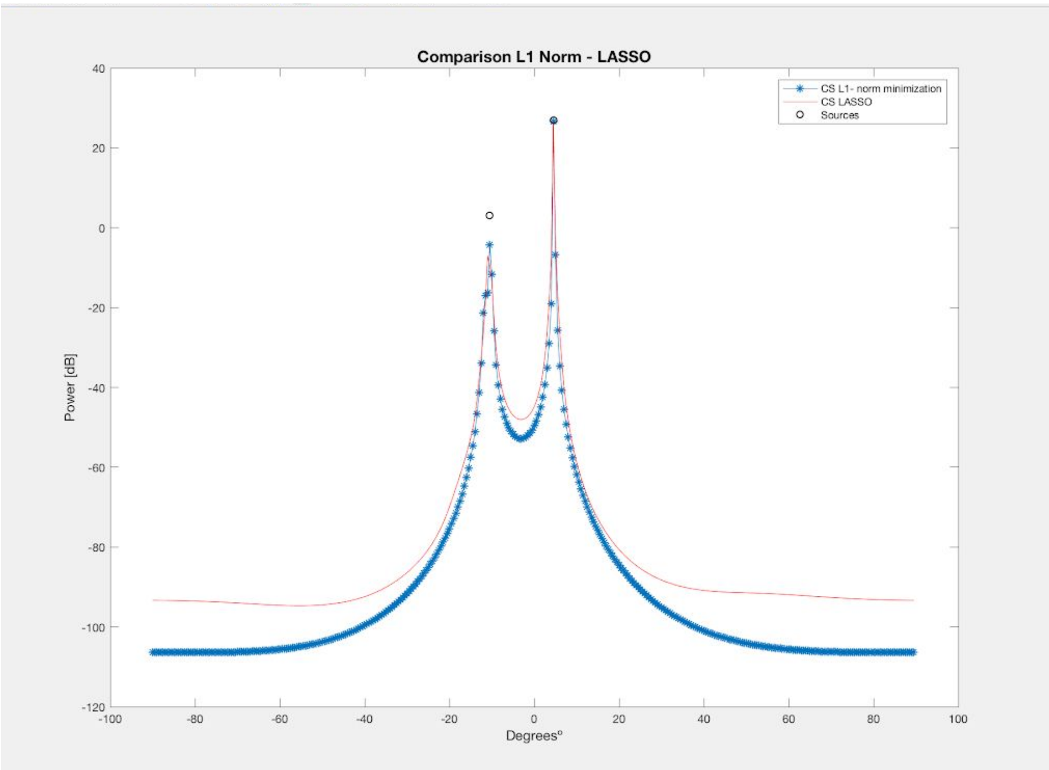


Fig7

The Figure shows how *Eq(6)* l_1 - norm minimization and complex LASSO *Eq(7)* are equal if we find an appropriate regularization parameter $\mu = 0.5$ for a existing value of noise floor $\varepsilon = 0.01$.

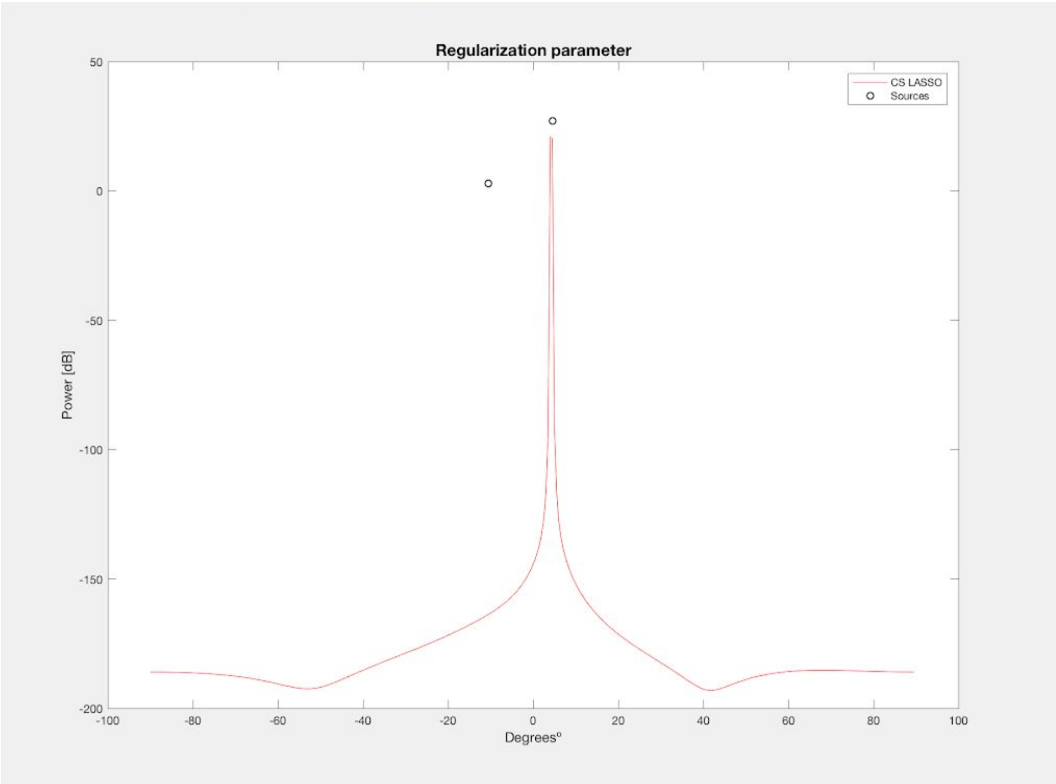


Figure 8

The c-LASSO *Eq(7)* algorithm when regularization parameter takes large values, $\mu \uparrow\uparrow$. The regularization parameter used in this simulation is $\mu = 30$. The image shows only one path of the received signal instead of two paths. Increasing the parameter has eliminated a significative coefficient. Now the DOA only have one path in the angular domain.

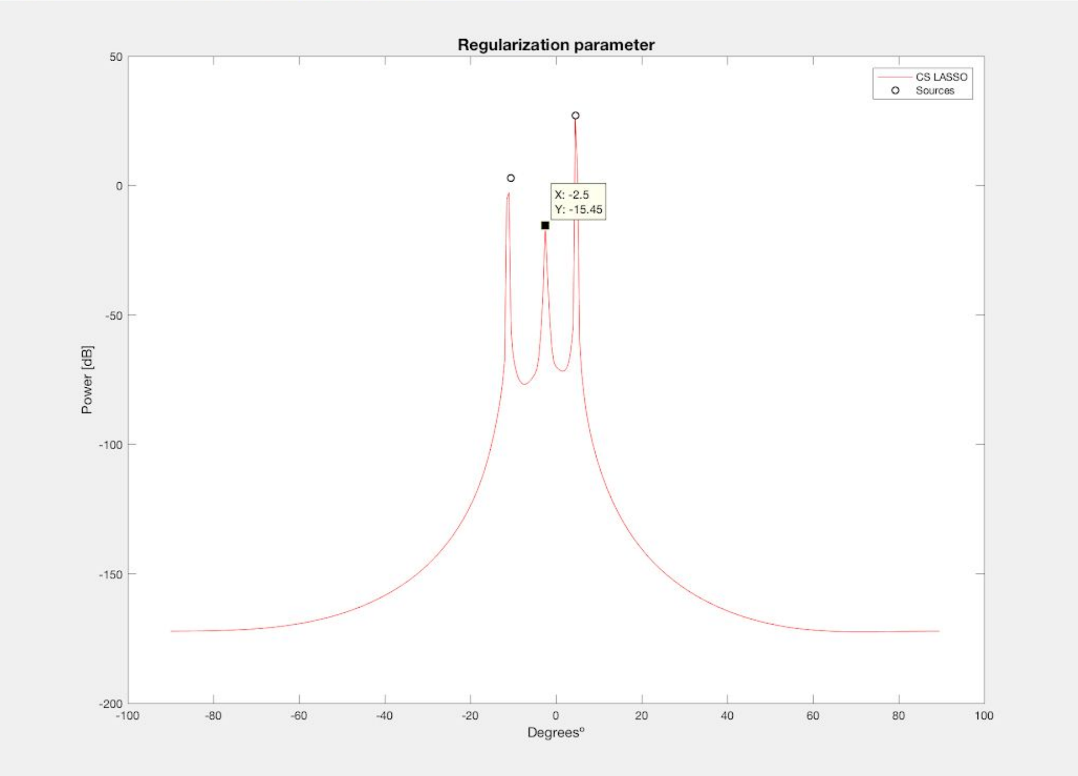


Figure 8

The c-LASSO *Eq(7)* algorithm when regularization parameter takes low values, $\mu\downarrow\downarrow$. The regularization parameter used in this simulation is $\mu = 0.8$. The image shows three paths estimated. There is a new source in -2.5º with -15.45dB. When the regularization parameter is low, the estimated vector is not sparse enough and detect coefficients that are not relevant.

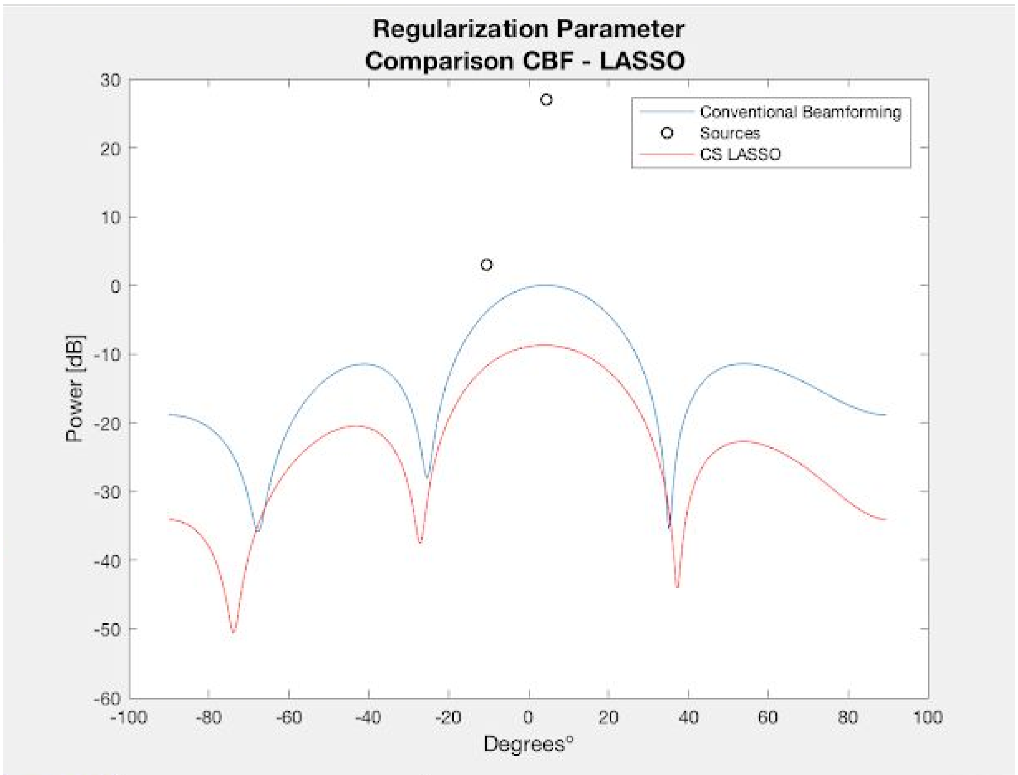


Figure 9

The c-LASSO *Eq(7)* algorithm when regularization parameter takes the zero value, $\mu = 0$, is related to the Conventional Beamforming.

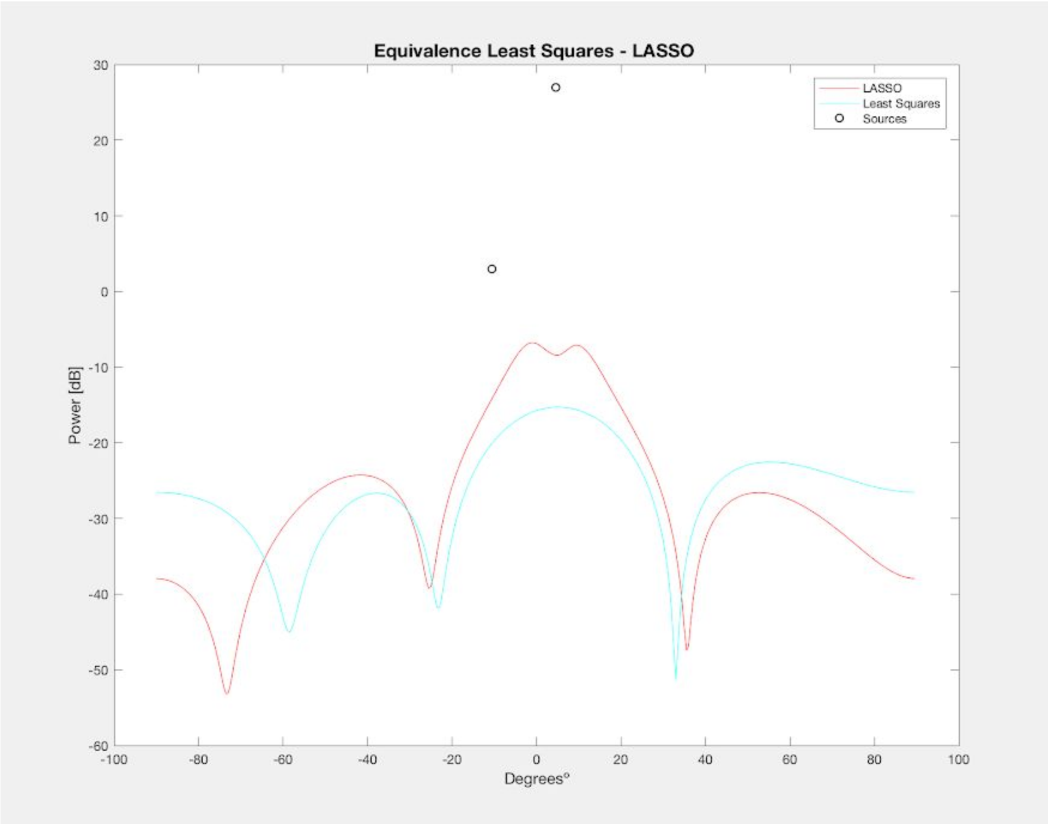
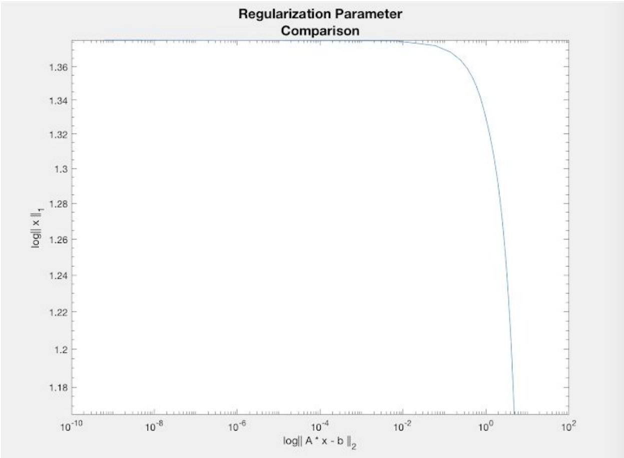


Figure 10

The c-LASSO *Eq(11)* algorithm and the Least Squares Eq(12).

Found a parameter $\delta_{LS} < \delta_{LASSO}$, $\delta_{LS} = 10$ and $\delta_{LASSO} = 70$.

Then we conclude $\widehat{x}_{LASSO} = \widehat{x}_{Least\ Squares}$.



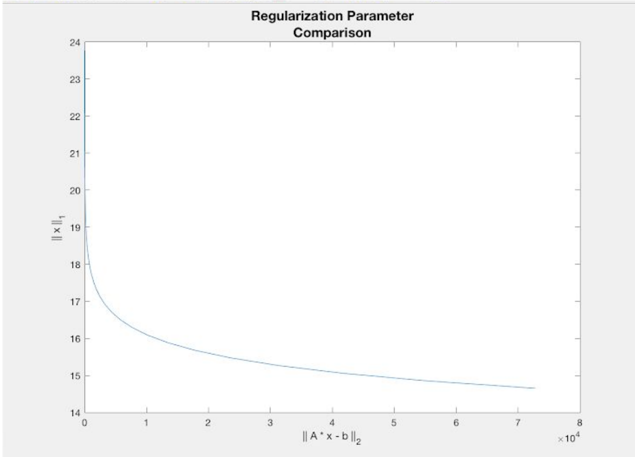


Figure 11a. Linear scale

Figure 11b. Log-log scale

The data error $\| y - A\hat{x} \|_2^2$, describing the goodness of fit, versus the l_1 -norm for the estimated solution $\hat{x}$ for different values of the regularization parameter μ in the c-LASSO problem *Eq(7).*

4.2. Channel measurements analysis

The acquired data by the receiver during the measurements in Seestadt has the following characteristic.

The bandwidth of our signal is 80 MHz, the Multi-tone Frequency Division Multiplexing (MTFDM) with 641 tones brings us a different channel matrix for each 641 different frequencies.

The 80 MHz Bandwidth signal is sampled in 641 tones and gives us a frequency

vector [5.7601GHz:100KHz: 5.8401GHz]. For the simulation, I have selected few frequencies and I worked with the corresponding Channel 4x4 Matrix.

4.2.1. Result 1

The results have the following description:

Transmitter antenna configuration number 1 and orientation number 1

Receiver antenna configuration number 1 and orientation number 1

Frequency selected 5.7666 GHz

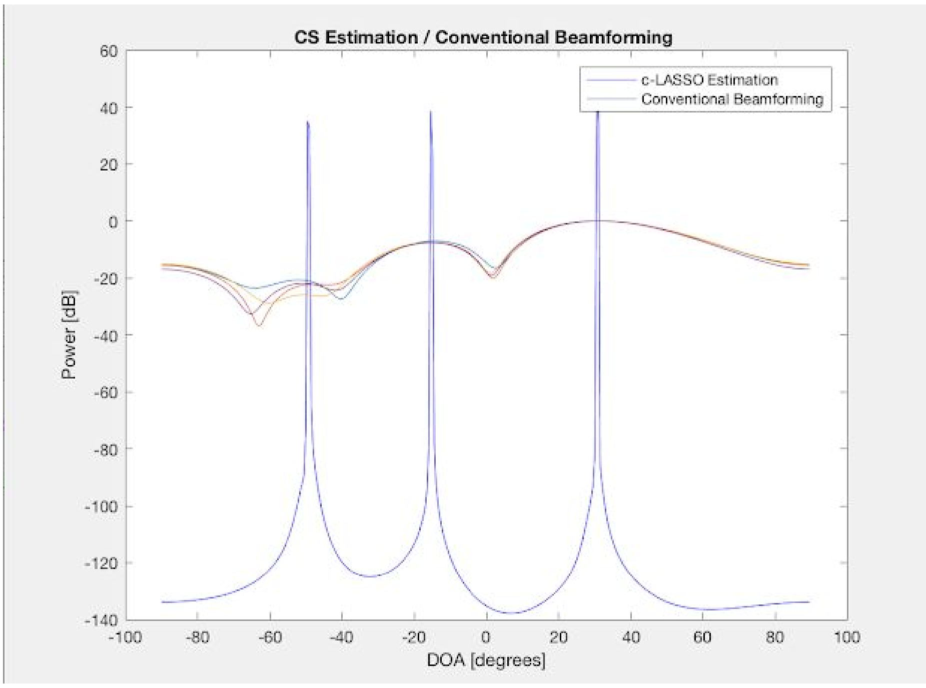


Figure 12. The c-LASSO algorithm as DOA estimator compared with the Conventional Beamforming.

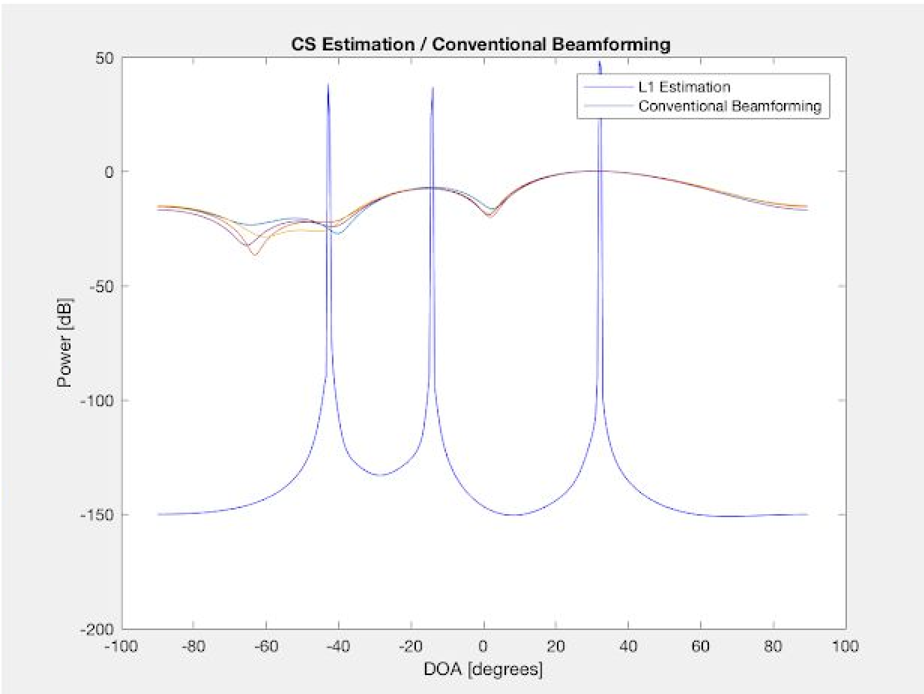


Figure 13. The $l_1 - norm$ algorithm as DOA estimator compared with the Conventional Beamforming.

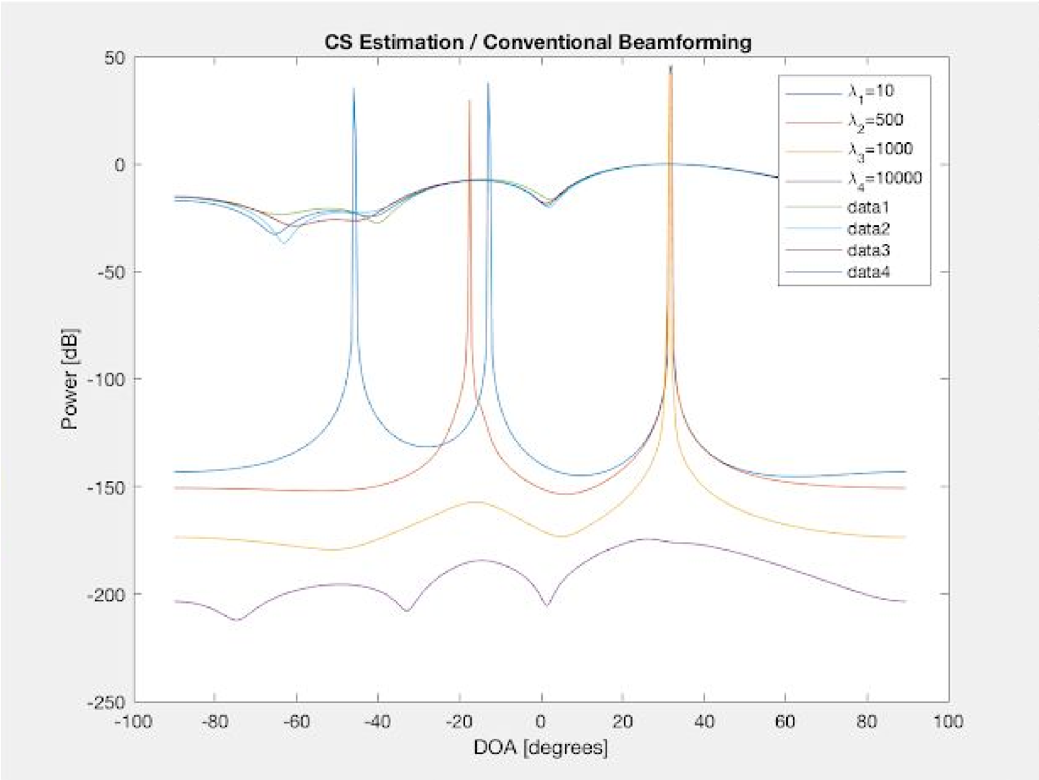


Figure 14. Different regularization parameters influence in the proper detection of the DOA.

4.2.2. Result 2

The results have the following description.

Transmitter antenna configuration number 1 and orientation number 2

Receiver antenna configuration number 1 and orientation number 1

Frequency selected 5.850 GHz

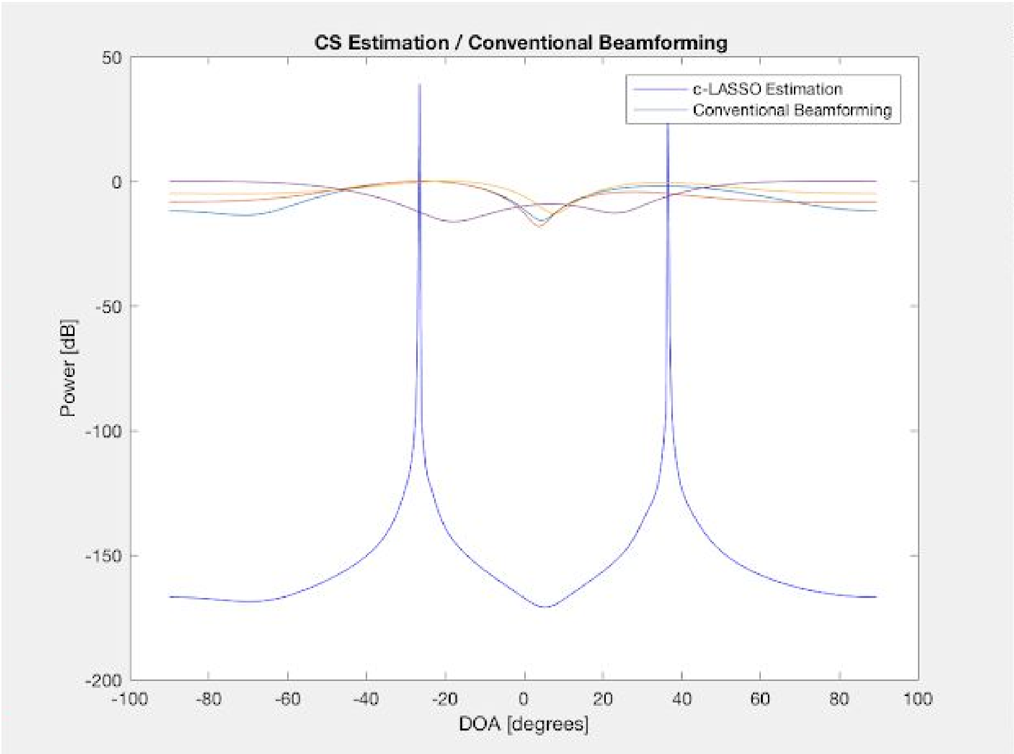


Figure 15. The orientation of the array antenna transmitter has changed and the receiver only can distinguish two paths of the signal.

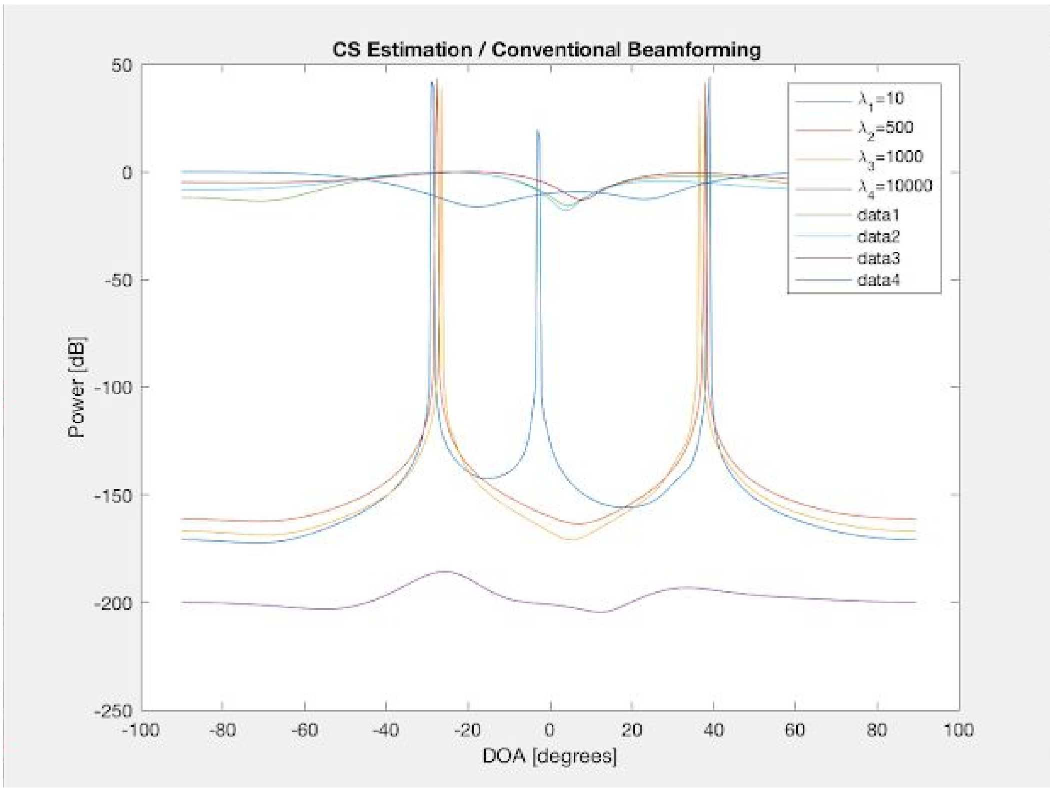


Figure 16. The regularization parameter influence in the proper detection of the DOA.

5. Budget

In this chapter the total budget for this project is computed. It includes the licence for the software used as well as the hours I dedicated to this project as a Junior Engineer.

5.1. Software

Matlab License: Free for Students, 500 € for educational institutions as UPC

5.2. Personal remuneration

<table>
<tr><th>Task</th><th>Subtask</th><th>Time</th><th>Cost/Hour</th><th>cost</th></tr>
<tr><td>Code design</td><td>Meetings with the professor</td><td rowspan="3">340 h
(17 weeks,
20h/week)</td><td rowspan="4">8 €/ hour</td><td rowspan="3">2.720 €</td></tr>
<tr><td>Code implementation</td><td>Meetings with the professor</td></tr>
<tr><td>Data analysis from the measurements</td><td>Matlab code</td></tr>
<tr><td>Deliverables</td><td>Project Proposal and Work Plan
Critical Review
Degree Thesis</td><td>130 h</td><td>1.040 €</td></tr>
<tr><td>TOTAL</td><td colspan="4">3760 €</td></tr>
</table>

Table 8

6. Conclusions and future development:

- Compressed Sensing is a new sampling technique that has better properties than the Nyquist rate.
- The complex LASSO as a solution of the non-convexity problem of the Compressed Sensing.
- The complex LASSO has been shown as a high-resolution Direction Of Arrival estimator compared with other traditional estimators like Conventional Beamforming.
- The importance of the regularization parameter selection for the proper detection of the Direction of Arrival.

Future Developments

1. Build the LASSO for multiple snapshot.
2. Implement Direction of Departure estimation with single and multiples snapshot.
3. Build a software to work with the LASSO algorithm in real time and implement it inside the receiver side.

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Glossary

AOA	Angle Of Arrival
AOD	Angle Of Departure
CBF	Conventional Beamforming
c- LASSO	Complex Least Absolute Shrinkage and Selection Operator
CS	Compressed Sensing
DOA	Direction Of Arrival
DOD	Direction Of Departure
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
LS	Least Squares
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MTFDM	Multi-tone Frequency Division Multiplexing
MUSIC	Multiple Signal Classification method
ULA	Uniform Linear Array